Here is the dynamic problem from the review session:

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Description automatically generated

Parameters: β

Exogenous variables: y, r

Choice variables: c

State variable: a

Utility function: u( . )

The problem needs to be solved over two-periods because the state variable (a) carries over from time t to time t+1. We do not need to solve other future periods. Notice that a\_[t-1] is already set at period t. This means by setting c\_t also sets a\_t (and vice versa):

A math equation with plus and minus signs

Description automatically generated with medium confidence

y\_t, r\_[t-1], and a\_[t-1] are all known at time t. The household thinks about how holding assets will affect its future budget constraint, so we have a constrained optimization problem:

A math equations with numbers and symbols

Description automatically generated with medium confidence

I specify log utility and write out the first two periods of the problem. All the other periods are ignored. I could have specified the problem with just two periods, but you will frequently see the infinite sum in more complicated models, and the solution is usually the same as I give here.

If you take first-order conditions, you will get the same result I give above. It should be clear you can substitute for λ\_t in the FOC for a\_t. What about λ\_[t+1]? Recognize you can iterate the FOC for c\_t forward one period

Once you put everything together, you get the Euler equation

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Description automatically generated with medium confidence

This identity tells us that if the interest rate (1 + r) is higher than 1/β, then the household will prefer to lower consumption today and wait until t+1 to consume. This is a starting point to think about the relationship between the interest rate and demand in an economy. This tells us how monetary policy can affect behavior (in the short-term). In the long-term, the interest rate needs to ensure that consumption remains sustainable. This becomes more intuitive when the interest rate is endogenous to the economy, a point we have yet to reach.